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Patent Application of

Bill R. Baker

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For

TITLE: COMPOUND PHASE ENCODING FOR PRINTED SERVO PATTERNS

CROSS-REFERENCE TO RELATED APPLICATIONS

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This application claims priority from U.S. Provisional Patent Application No. 60/447,442 filed Feb. 14, 2003, the entire disclosure of which is incorporated herein by reference, and International Application No. PCT/US 03/41733 filed Dec. 31, 2003.

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FEDERALLY SPONSORED RESEARCH

Not Applicable

SEQUENCE LISTING OR PROGRAM

Not Applicable

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BACKGROUND OF THE INVENTION

1. Field of the Invention

30 This invention relates to printing or rapidly transferring fixed servo reference data to surfaces such as those of magnetic storage media and, more particularly, to printing servo data with compound phase patterns that provide absolute position information, that are well suited to commercial data channel chips, and that require relatively little space on the recording surface.

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2. Description of the prior art

Modern magnetic recording systems have servo information or position markers written in an interleaved fashion on the same surface on which data are recorded. To simplify the system the same head is used to read the user data and the servo information. Various special formats
40 are used for the servo information to enable using measurements and subsequent signal processing to determine the position of the read head relative to the center of the desired data track. A head movement mechanism and servo control system keep the read head close enough to the data track center to assure reliable reading and writing of user data.

45 Early hard disk systems used a special machine, a servo track writer (STW), e.g. U. S. Patent 5333140, to record the servo information on the disk surface. The STW includes a clamping system to hold the hard disk drive (HDD) in a fixed reference position and an external motor with a laser or optical encoder to accurately move a reference pin that extended into the HDD. The actuator or head positioning mechanism of the HDD is biased against the pin so
50 the write head of the HDD can be placed at any desired radius by moving the external motor according to its encoder system. The STW also includes a clock head that is temporarily placed on a surface of the disk by means of a special aperture in the HDD case. Circuitry of the STW writes a clock signal or timing reference by applying a pattern of write current to the clock head. The clock head reads the timing reference signal as the HDD write head is
55 moved to any desired radius. Since the timing reference is fixed relative to the disk it is possible to write servo patterns of a desired form as function of radius and angle on the disk. Related STW are used for removable media such as the ZIP floppy diskette system manufactured by Iomega Corporation.

The STW must turn the disk through at least one revolution to write the servo information,
60 and another fraction of a revolution is required to move the head to the next radius. Many
servo patterns use the edges of special bursts or sub elements in the determination of the
position from the read back signal, so it is necessary to write the servo bursts at radial
displacements of a fraction of the data track width. Therefore it is usually required to write
two or more servo tracks for each data track. Some HDDs now have more than one hundred
65 thousand data tracks, so it may require tens of minutes to write the servo pattern.

Because the enclosure of the HDD is required to have openings for the clock head and for the
reference pin it is necessary to use the STW in a special clean room to avoid contamination of
the head-media interface. It is expensive and difficult to maintain the complex STW in such a
70 clean environment.

Another method uses a Media Writer, a special machine that writes a servo pattern on ten or
more disks on a special spindle. Then disks are removed from the spindle and placed into
HDDs. That method also requires about an hour to write the pattern, and the special
75 equipment must also be operated in a clean room.

A different "printing" approach was proposed by Bernard, et al., in U.S. Patent 3869711,
1975. In that method the desired servo pattern is replicated in a master disk consisting of a
nonmagnetic substrate about one millimeter thick with embedded strips of a highly permeable
80 material such as cobalt. The face of the master containing the cobalt elements is placed in
contact with a D.C. erased slave disk. Then a permanent magnet producing an oppositely
directed field is brought close to the back surface of the master and is rotated one revolution
relative to the master-slave pair. The cobalt elements shield portions of the slave disk leaving
them in the original D.C. state, but gaps in the cobalt pattern allow the field to penetrate. The
85 field is concentrated at the gaps and the increased fringing components reverse the
magnetization of the adjacent portions of the slave disk.

That method was recently advanced by the development of automated "disk printing"
machines, see Ishida, T., et al., "Printed Media Technology for an Effective and Inexpensive
90 Servo Track Writing of HDDs", IEEE Trans. Magn., p1875, 2001 and Sugita, R., et al., "A

Novel Magnetic Contact Duplication Technique for Servo-Writing on Magnetic Disks”, IEEE Trans. Magn. p2285, 2001. The rapid transfer of the pattern to the entire surface of the slave disk is done as the last step at the end of a conventional disk manufacturing line.

95 Important feature sizes, typically line widths and the thickness of cobalt elements, have been steadily decreasing, but the transition density of printing initially lagged that of conventional write heads. Direct printing of a conventional edge of burst servo pattern resulted in poor performance at contemporary data densities, (Ishida, et al., “Demodulation of servo tracking signals printed with a lithographically patterned master disk”, *IEEE Trans. Magn.*, Vol. 37,
100 No. 4, Jul., 2001, pp1412-1415).

The conventional Gray codes would provide the absolute radial position, but they are not well suited to the printing process and result in fuzzy transitions at track edges. Unfortunately those codes occupy a large part of the surface, they are not well adapted to the architecture of
105 commercial channel chips, and they require larger variations of widths of lines and spaces. The large variations of feature sizes exacerbate noise sources of the printing process. Proper choice of geometry including width and thickness of the cobalt elements and width of the gaps is necessary to assure magnetic switching of the slave medium next to the gaps of the master without saturating the cobalt film to produce “secondary gaps” and consequent writing
110 of spurious pulses or noise (Saito, A., et al., “ Magnetic printing technique for longitudinal thin film media with high coercivity of 6000 Oe”, J. Appl. Phys., V 91, p 8688, 2002 and Baker, “Tradeoffs for magnetic printing of servo patterns”, J. Appl. Phys., p8691, 2002).

Therefore it was proposed in U.S. Patent 6304407 to use the printed pattern as a reference
115 system for self-servowriting (SSW). Because the printing method involves several processes such as optical diffraction, diffusion in the photo resist, and shadowing during sputtering of the cobalt, it is difficult to produce square corners or small radii of curvature at the ends of the cobalt lines. Phase methods were used as in U.S. Patent 3686649 for the position information, and the phase was measured by discrete Fourier transforms (DFT) in the manner
120 of U.S. Patent 5784296. In this method the ends of the bars in the inclined phase elements are excluded from the sample window, and pulses are measured at the long, clean edges of magnetic transitions.

After assembly of the HDD it was removed from the clean room and placed on a self-test rack where it began its self-servowriting directed by the embedded firmware. Well-known self-test methods measured possible pattern eccentricity and any minor errors of the position information for each servo block printed on the disk. Then corrections were applied for subsequent writing of a final servo pattern. The relatively low additional cost of printing one surface of a disk eliminated the need for an expensive STW and the clean room in which to operate it.

Mitsubishi Corporation using well-known magneto-optic (MO) techniques subsequently developed an alternate printing method. The same pattern described above was first replicated as a chromium on glass reticle or mask. Such masks are commonly created in photolithography. Opaque features are left as a thin chromium film on a transparent glass substrate. In this case too the slave disk is initially D. C. erased, and the printing magnet applies a field in the opposite direction. The applied field is a bit lower than the coercivity of the slave so its magnetization is preserved until a laser is flashed through the mask. The brief laser pulse heats areas under gaps of the pattern. The surface temperature of the recording film rises quickly decreasing the local coercivity and allowing the selected regions to switch magnetization directions.

This contact magneto-optic method also has limitations due to diffraction and to the difficulty of maintaining the small space between reticle and slave disk and due to reflections between the two. Using an antireflective coating on the reticle is difficult because the energy density of the laser irradiation damages the coating. Some of these problems are ameliorated by a projection printing demonstration (Wang, L., et al., "Photo thermal patterning on magnetic media", J. Appl. Phys., V 91, p8685, 2002).

As is well known in the disk drive industry the number of servo wedges or position bearing segments of the disk must increase as the track density increases. Drives now have a few hundred wedges and the trend is toward higher densities. New processes such as Nano Imprint Lithography can now make finer patterns.

155 The SSW process described above utilized early printed disks when the critical feature sizes or printable line and space widths were greater than one micron. There was no absolute position information, but it was adequate to slowly move the head in small steps along the radial extent of the HDD to write each of the final servo tracks. It was not possible to move rapidly to a desired radius. The relatively large printed reference patterns were simply overwritten after the final pattern was completed.

160 It is also well known that fields at the edges of conventional write heads are poorly controlled, (Van Herk, "Analytical expressions for side fringing response and crosstalk with finite head and track widths", *IEEE Trans. Magn.*, Vol. 13, No. 6, Nov., 1977, pp1764-1766 and Tsang, et al., "Disk-noise induced peak jitters in high density recording", *IEEE Trans.*
165 *Magn.*, Vol. 29, No. 6, Nov., 1993, pp3975-3977.) Therefore the STW and the SSW and Media Writer methods all introduce noise at edges of bursts of conventional final servo patterns. That noise becomes a greater problem at higher track densities. Furthermore those prior methods all depend upon a servo system to position the head as each track is written, and any errors of that system will be passed on to the final servo pattern. Such errors,
170 including those induced by vibrations, may cause encroachment into adjacent tracks. On the other hand greater control can be devoted to the one-time production of the master used for printing methods.

175 BRIEF SUMMARY OF THE INVENTION

One object of the present invention is to provide convenient absolute position information from a printed pattern so the cross track position of a read sensor of a magnetic recording system or other track oriented machine can be verified from only the local data.

180 Another object of this invention is to improve magnetic printing by reducing variations in feature size and to better match the pattern to the channel chip capabilities.

Another objective is to reduce the area required for the printed pattern so that it can be used directly as the final servo pattern without rewriting in a SSW mode.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a simplified schematic diagram of a printed pattern of the prior art.

FIG. 2 is an attempt to print Gray codes according to prior art.

190 FIG. 3A is a diagram in logical coordinates that defines phase encoded position elements of the present invention.

FIG. 3B is an example diagram in logical coordinates of fine position and absolute position phase elements.

195 FIG. 3C shows initial phases Φ_A and Φ_{B0} in the standard interval with an intersection at $\pi/2 - \Theta$.

FIG. 3D shows normalized phases with $\Phi_B = \Phi_{B0} + \Theta$ and intersections at $\pi/2$ and $3\pi/2$.

FIG. 3E shows the difference $\Phi_0 = \Phi_A - \Phi_B$ normalized to the standard interval.

FIG. 4A is a diagram of the phase differences of A and B fine elements of Fig. 3B.

FIG. 4B is a diagram of phase variations of the higher order C burst of Fig. 3B.

200 FIG. 4C is a diagram of phase variations of the higher order D burst of Fig. 3B.

FIG. 5 shows phase variations of the C and D bursts vs the cross track position y.

FIG. 6 shows phase variations in an example positional value phase system.

FIG. 7 shows a system with reduced features to improve surface utilization efficiency.

205 **DETAILED DESCRIPTION**

The invention comprises two or more phase-encoded subsystems combined in a novel manner that provides a unique absolute position coordinate from local data. The resulting pattern is well suited to available methods for printing servo patterns on magnetic recording
210 media such as disks or linear tapes. The servo pattern can be easily read and decoded using commercial R/W (Read Write) or channels chips. This method could be used as reference for SSW (Self Servo Writing), but it requires much less surface area, and it eliminates track edge noise and track squeeze, so it can be used as a better, final servo pattern.

215 The greatly simplified schematic diagram in logical coordinates of Fig. 1 shows the magnetization distribution of printed patterns of the prior art. The abscissa or horizontal coordinate x is a down-track coordinate proportional to the time at which the read head

encounters an element of the pattern. The ordinate, y , is a cross-track coordinate which changes as the head moves to a new track. The white background of the diagram corresponds to a "background" state of magnetization directed generally in the longitudinal direction of the tracks, either toward the right or toward the left (positive or negative x direction). The crosshatched pattern elements are regions where the magnetization has been reversed by the printing process.

The head and read channel produce a pulse as the head crosses a transition between the two states of magnetization. The sign of the pulse when the head leaves the background state is opposite that created at the return to the background. The absolute sign of the pulse at exit from the background is not critical to this invention, and most modern read channels can be configured to work with any consistent background state.

For a linear tape system with a linear actuator that moves the read head straight across the tracks, Fig. 1 resembles the magnetization distribution in physical coordinates on the tape.

For a disk drive with a linear actuator the servo pattern of Fig. 1 would map onto a wedge shaped region in physical coordinates so that vertical lines of Fig. 1 would diverge for increasing radius. For a disk with rotary actuator the constant time lines of the servo wedge would also be arcuate as well as diverging for larger radius.

The servo block of the schematic diagram of Fig. 1 comprises elements 10 that are centered relative to the data tracks. Those track centered data include elements for clock synchronizing, certain special flags such as a SAM (Servo Address Mark), an Index and other markers and Gray code bits, 18, for coarse position information. Conventional edge of burst fine position elements 14 are offset from the track centerlines. The dashed lines suggest boundaries of the user data tracks. Generally the number of lines and spaces in the timing segment 10 would be greater, but this diagram shows the concepts in a greatly simplified form.

Demonstrations of demodulation using this prior art, (*ibid.* Ishida), were made at relatively low track density. The magnetic printing from a permeable film pattern does not work well

250 for Gray codes, 18, where some small regions should be reversed in the middle of the surrounding region of the background magnetization state. During printing an external field is applied in order to switch the regions under the openings of the pattern. However, the magnetic flux tends to stay in the permeable film of the master in large areas such as 18. Flux flows around smaller openings in the film leaving adjacent portions of the slave surface
255 unaffected.

Another problem is that large film areas, required in the usual Gray codes, may become saturated and lose their shielding characteristic. That allows flux to escape the film and create unwanted pulses or noise. At large gaps in the film, 16, the fringing field is smaller, and the
260 magnetization may not be completely reversed.

In the prior art special Gray code structures such as Fig. 2 have been studied. In that format every Gray code cell, of length L in the down track direction, is forced to have one line 22 and one space 20. Information is encoded by the presence or absence of a reversal in the
265 intermediate interval extending across the width of the track in the y direction. Unfortunately, the recording at the cross track ends of those information bits is poorly controlled and leads to noisy read back signal. That corresponds to the weakened field and corresponding noise found at the edges of conventional write heads, (ibid. van Herk, Tsang). A further difficulty of patterns such as those of Fig. 2 is that channel chips usually are binary based so Gray code
270 cells must contain 2 or 4 subintervals; three subintervals are not usable. Gray codes require accurate control of the sample clock and its phase.

The present invention avoids the problems of Gray codes by using phase encoding methods to provide absolute position information. This information is decoded from signals read at
275 any cross track location. It is not required to accumulate data as the head is moved incrementally from a special reference position such as a crash stop. Avoiding Gray codes allows simplification of the electronic circuits used to read the position information.

Fig. 3A illustrates important geometrical details of the phase encoding method. The pattern is
280 composed of lines and spaces of magnetization of alternating direction. For most printing methods the width, w , of such lines must be larger than the limit of the fabrication method,

and that limit is observed in the present method. The forward sloped lines, 30, and the backward sloped lines, 32, are arranged periodically in both x and y directions so that the down track period λ is constant in the logical coordinates. Generally the ends of such lines are difficult to print so the signal is sampled or read over a restricted interval such as A for the forward slanted group of lines. That group will be called the A burst in agreement with terms commonly used in servo systems for disk drives.

A group of backward slanted lines is designated as the B burst in Fig. 3A. In general it is convenient, but not necessary, to make the B burst a reflection of the A burst relative to a central vertical timing line as shown here. For the given configuration, signals from the A burst and B burst will be one half cycle out of phase where edges of the slanted strips cross the vertical dotted lines. It is always possible to mathematically offset phases of signals read from the bursts to generate any desired phase relations between them. The preferred method of demodulating the read back signal uses a channel chip with DFT. The resulting sine and cosine components of the Fourier transform are quadrature components for each burst, and a conventional four-quadrant arctangent transformation yields traditional phase angles. Some truncated lines are omitted in the lower part of Fig. 3A to make the geometry more clear, but generally the elements will fill the entire transverse range of the pattern as suggested in Fig. 3B.

The preferred embodiment uses the DFT of the channel chip to obtain the sine and cosine transforms of each burst such as A, B, C, and D of Fig. 3B. Other bursts may be added to the pattern, and many channel chips accommodate six bursts or more. Other methods can be used to measure timing delays of the various bursts, but the DFT is readily available. Sophisticated channel chips made for tens of millions of disk drives are very cost effective and can also be used for tape systems if they also function at lower frequencies.

Well known methods are used to compute the phase delay of the signals from each burst relative to the sample clock that is aligned, as usual, with timing features of the first part of the servo pattern. For the example illustrated in Fig. 3B the phase angles of the four bursts would be Φ_A , Φ_B , Φ_C , and Φ_D .

It can be seen from Fig. 3A or Fig. 3B that Φ_A , the phase delay of the A burst, will generally
 315 increase or be shifted in the x direction if the read head moves in the y direction. From the
 antisymmetric geometry it follows that Φ_B , the phase delay of the B burst generally decreases
 at the same rate. Therefore using the difference of those phases to measure the cross track
 motion can double the sensitivity. For the reflected geometry the SNR (Signal to Noise Ratio)
 would be increased by about a factor of $\sqrt{2}$ if the noise sources were independently
 320 distributed.

The period of the antisymmetric bursts, A and B, of Fig. 3A is twice the height h of a line in
 the y direction. However, the phase difference, $\Phi_0 = \Phi_A - \Phi_B$, has a period or pitch, p_0 , only
 half as big as shown in Fig. 3B. This method of compound phase encoding can be described
 325 by using various equivalent mathematical notations in addition to the following method.

Generally the phases Φ_A and Φ_B are adjusted by adding or subtracting multiples of 2π to give
 a value in the fundamental interval $0 \leq \Phi < 2\pi$, and their discontinuities are offset. With no
 loss of generality a simple test and conditional addition of an offset maps Φ_0 to two of the
 330 fundamental cycles, each corresponding to motion of p_0 in the transverse direction. The
 example of Fig. 3C shows initial phases Φ_A and Φ_{B0} in the standard interval with an
 intersection at $\pi/2 - \Theta$. Adding Θ to Φ_{B0} and normalizing to the standard interval gives Φ_A
 and Φ_B with intersections at $\pi/2$ and $3\pi/2$ as in Fig. 3D.

335 Because of various noise sources in every recording system there may be a small error in the
 sample clock and that would create an error in each of the phase values. Those errors would
 cancel in the difference, $\Phi_0 = \Phi_A - \Phi_B$, to reduce effects of timing jitter. The value of the
 error can be found from the sum, $\Phi_S = \Phi_A + \Phi_B = 2\delta$. It can be seen from diagrams such as
 Fig. 3D that the conventional average of Φ_A and Φ_B would generally give one of two values,
 340 $\pi/2$ or $3\pi/2$ for this example. However, the normalized sum Φ_S lies in the standard interval,
 and half its value gives the smaller result, $\delta = \delta_L = \pi/2$.

The measured error, $\tau = \delta - \pi/2$, can be used to correct the measured phases of higher order
 bursts such as C, D, etc. that follow the fundamental A, B pair of bursts provided τ is smaller

345 than $\pi/2$ in magnitude. For systems that have even greater clock jitter it is possible to include a special higher order burst with such a large crosstrack pitch that its elements are essentially parallel to the timing elements. Variation of the phase of such a special burst provides correction information for clock offsets even greater than a full cycle provided all samples are taken within valid phase bursts.

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Bursts C, D, etc. are made with various pitches or periods in the transverse or y direction, but with the same period, λ , in the down track or x direction. The description and application is simpler when the crosstrack pitches of higher order bursts are integer multiples of the fundamental pitch p_0 as in the following.

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The simple illustrative example of Fig. 3B includes a C burst with transverse pitch $p_1 = 3 p_0$, where p_0 is the pitch of the difference, $\Phi_0 = \Phi_A - \Phi_B$. The D burst in this example has pitch $p_2 = 4 p_0$. It is convenient to make the sample windows of the A and B bursts equal, but it is generally possible to decrease the widths of higher order bursts used for absolute position
360 determination.

As shown above, a constant can be added to the phase of each higher order burst to assure that they are all zero at a certain zero of Φ_0 . That common zero location is a convenient starting point for a cross track coordinate.

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Measurements have shown that the phase of such bursts can be measured with errors smaller than about one per cent of a cycle of 2π radians. Variations of the phases are shown in polar form in Fig. 4 as the head is moved in the y direction over a distance about 13 times the pitch p_0 of the fundamental A, B pair. Fig. 4A shows variation of the fundamental difference, $\Phi_0 = \Phi_A - \Phi_B$, and the integral cycles are marked with numbers 0, 1, 2, ..., 12, 13, ... at angle zero. Fig. 4B plots Φ_C for the same motion, and numbered markers indicate the phase of the C bursts at the start of each cycle of the fundamental difference. The pitch of the C burst is three times that of the fundamental difference so Φ_C advances at one third the rate of Φ_0 . Similarly Fig. 4C shows Φ_D which advances at one fourth the rate of Φ_0 .

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Exaggerated crosshatched sectors in Fig. 4 suggest the errors or uncertainties in each of the three phase measurements at positions corresponding to zero phase of the fundamental Φ_0 . It is sufficient that the error regions of the higher order phases, Φ_C and Φ_D , do not meet. Then it is always possible to uniquely determine the cycle number of the fundamental phase, Φ_0 , and hence the absolute position for all fundamental cycles 0 to 11 = 3*4-1.

As in all such measurements it is possible to determine the phase of each burst or pair of bursts only within one revolution. With no loss of generality the phases can be constrained to the interval $0 \leq \Phi < 2\pi$ after conventional phase shifts to align bursts as shown in Figs 4, 5, and 6. A method to select the appropriate cycle of the fundamental phase is explained by Fig. 5 for the same example of Figs. 3B, 4A, 4B, and 4C. The higher order phases, Φ_C and Φ_D , are plotted versus the transverse displacement y normalized relative to the fundamental pitch p_0 . All three phases are equal to zero at the start, but the higher order phases change at their different rates. Challenges occur when the fundamental phase Φ_0 jumps from a value near 0 to a value near 2π . Those jump points are marked by the row of integers, j , corresponding to the number of the cycle beginning at that value of the displacement y .

At those locations the phase Φ_C also jumps from one to the next of its segments of one-third of a rotation, and Φ_D jumps to its next segment of length one-fourth of a rotation.

As suggested by the crosshatched sectors of Fig. 4, small errors may occur in the measurements, but the ambiguity is resolved by first deciding the value of the fundamental phase Φ_0 with its possible, small error. If Φ_0 is found to be zero or slightly larger, then Φ_C and Φ_D are both chosen to be in the segments corresponding to values of y just larger than that at the ideal jump point. Cases where Φ_0 is slightly less than 2π results in segment numbers for C and D phases corresponding to y values just under those at the ideal jump point.

The decision making process is similar to that commonly used in reading multi-dial meters having separate dials for units, tens, hundreds, etc. A simple example is used to read an analog clock when the minute hand is near 12. The formal method is:

- 1) decide the value of minutes
- 2) add (1/2-minutes/60) hour to the apparent position of the hour hand
- 3) decide the value of the hour

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Similar methods in the servo code yield a position coordinate over the radial span of the disk pattern with the resolution of the fundamental difference Φ_0 .

415 Because the phase angles, Φ , are confined to the range $0 \leq \Phi < 2\pi$ it follows that the integers designating the subintervals are also limited and repeat modulo the number of segments. The number of phase segments for a burst is the ratio of its pitch to the fundamental pitch, p_0 . It is necessary to decode the combinations of segment indices of the C and D bursts in order to find the cycle number of the fundamental A, B difference.

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In Fig. 5 the markers indicate the start of each segment, and the rows of integers under the horizontal axis correspond to the segments of Φ_0 , Φ_C , and Φ_D respectively. The index k for the C burst repeats modulo 3, and index m for the D burst repeats modulo 4. Because the integers 3 and 4 are relatively prime (have no common factors) the combinations of k and m are unique over the first $3 \times 4 = 12$ fundamental cycles. Therefore each of the first 12 k - m pairs identify a unique fundamental cycle numbered j where j is contained in $[0,1,2,...11]$. For larger values of j the k - m pairs would repeat.

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The values of k and m are computed from the phase measurements, and then the value of j can be computed for this example from the following algorithm.

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ALGORITHM 1

$$j = M * \text{mod}(k - m + K * M, K) + m$$

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Here the modulo function **mod** is the remainder obtained by dividing the first integer argument by the second integer argument in parentheses. Algorithm 1 is valid for any two relatively prime numbers **K** and **M** where $M > K$ and $\text{mod}(M, K) = 1$. Those conditions are
 440 satisfied by any pair of successive integers, that is, with $M = K + 1$, as well as by many other pairs.

The following decoding algorithm is valid if $\text{mod}(M, K) = 2$:

ALGORITHM 2

445 $r = \text{mod}(m - k + K, K)$
 if $r = 0$
 $t = 0$
 else if r is even
 $t = K - r/2$
 450 else
 $t = (K - r) / 2$
 end
 $j = t * M + m$

455 Additional bursts can be included in the compound phase pattern to extend the length of the unique combinations of the sub indices provided the number of values for each index has no common factor with any other corresponding number. For example $K = 3, M = 4, N = 5$ would give unique codes k, m, n for j running from 0 to $3 * 4 * 5 - 1 = 59$.

460 It can be shown that an odd integer and the next two integers have no common factor, so such a triple can be used to form a unique code sequence of length equal to their product. Then the previously described algorithms can be combined to decode the triplets as follows.

ALGORITHM 3

K is odd
 465 $M = K + 1$
 $N = M + 1$
 $r = N * \text{mod}(m - n + M^2, M) + n$

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s = mod( r - k + K, K )
if s = 0
470     t = 0
else if s is even
        t = K - s / 2
else
        t = ( K - s ) / 2
475 end
j = t * M * N + r

```

Some examples of the successive triplets described above are:

	K	M	N	K*M*N
480	5	6	7	210
	11	12	13	1716
	19	20	21	7980
	51	52	53	140556

Many other combinations of relatively prime integers can be found and similar algorithms
485 can be set up to decode the corresponding sets of segment numbers to find the index j of the
fundamental cycle. Then the fraction of the fundamental cycle can be added to compute the
transverse coordinate as

$$y = (j + \Phi_0 / (2\pi)) * p_0 \quad (1)$$

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PREFERRED EMBODIMENT

The previous methods using relatively prime numbers of sub intervals could be decoded more
easily when the numbers of sub intervals were about the same for all higher order bursts.

495 However, errors in measurements of the phase angles are nearly independent of the pitch of
the bursts, and that allows use of an encoding method similar to positional number systems
such as our conventional decimal notation.

As a simple example let the first higher order burst have a pitch R times greater than the
 500 fundamental pitch p_0 . Then R sub intervals can be accurately identified in each cycle. Let the
 pitch of the next burst be $R * R$ times p_0 . For each additional more significant burst let its
 pitch be R times that of its predecessor. The 2π phase range of each burst can be accurately
 divided into R equal phase segments that each identify one cycle of the preceding, less
 significant burst. If there are Q of such higher order bursts, and if the segment indices, k_q , are
 505 measured for each burst, with $0 \leq k_q \leq R - 1$, then the cross track position, y , can be
 computed as

$$y = [k_{Q-1} * R^{Q-1} + k_{Q-2} * R^{Q-2} + \dots + k_1 * R + k_0 + \Phi_0 / (2 \pi)] * p_0 \quad (2)$$

510 This absolute position can be mapped in various ways to define the physical spacing of tracks
 in the transverse direction. For example, in hard disk systems some heads may function better
 than others, and regions near the inner radius of a disk may perform differently from regions
 near the outer radius. The final, physical mapping or track spacing can be adjusted to
 optimize robustness and capacity of the drive.

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Fig. 6 shows a simple example of such phase variations versus transverse coordinate y . The
 radix, or ratio R , is 4 in this case. The servo control computations are often done with a
 binary computational unit or DSP (Digital Signal Processor) operated in integer format.
 Some calculations such as multiplication and division by R can be done with simpler shift
 520 and add operations if R is a power of 2.

In Fig. 6 the segment indices are in the rows designated as k , m , and n , and the index of the
 fundamental cycle or the integer part of the quantity in square brackets in equation 2 is

$$525 \quad j = k * 4^2 + m * 4 + n \quad (3)$$

Of course, the number of phase segments could be changed for each burst, and then the
 position values would not be simple powers of a single ratio R , as in equation 2, but would be
 the product of ratios of all lower order bursts.

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In any case the ambiguity caused by the usual system noise at each segment boundary is resolved by working from the phase of the fundamental burst upward through each more significant burst, as explained above. That results in a total error no larger than that of the fundamental burst.

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For track oriented servo systems the on-track position must be maintained with deviations no greater than a small fraction of the nominal track pitch. Therefore narrower tracks require more servo information blocks, and that reduces the area available for user data or other purposes.

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The compound phase system of the present invention offers a means, as suggested in Fig. 7, to ameliorate the space requirements for servo information. Some of the servo blocks comprise more elements such as a synchronization segment 60 for a timing PLL (Phase Locked Loop), a SAM (Servo Address Mark) 62, certain flags 64 for index or other markers, a forward fine burst 30, a backward fine burst 32, and higher order position bursts 70, 72, and 74. Such complete servo blocks can be combined with the user data areas 78 and with abbreviated servo blocks such as those shown comprising only the fine position bursts 30 and 32. Modern channel chips such as those made by Marvell Corporation support the shorter servo blocks.

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Effective disk drive or tape transport control systems may use timing signals recovered from the servo blocks to regulate the motion of the medium and to synchronize the servo readback clock to the recorded pattern. At start up the information retrieved from the complete servo blocks can be used to initialize the timing systems. When the medium speed is nearly correct then the delay δ of the fundamental bursts of the abbreviated blocks can also be used for final tuning of the timing.

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Generally the servo pattern may be slightly eccentric relative to the axis of rotation after a printed disk is assembled into a drive. Well known self-test methods can be used to measure the eccentricity of the pattern, and the final data tracks can either be computed to form concentric tracks or the tracks can follow the eccentricity of the printed servo pattern by using standard feed-forward methods. In either case predictions of the control system can be

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verified and corrected at each of the fundamental or fine position chevrons. Adding one or more bursts of higher significance to the reduced servo blocks can enhance the robustness or
565 tolerance of the system to minor disturbances.

During rapid seek motion an advantage of the positional encoding system is that it is only necessary to decode the more significant bursts of the complete servo blocks. As the transverse seek speed decreases it is easier to decode the less significant bursts, and finally in
570 settling and tracking modes the fundamental or finest bursts are decoded and used in the control system.

In addition to the above mentioned examples, various other modifications and alterations may be made without departing from the invention. Accordingly, the above disclosure is not to be
575 considered as limiting and the appended claims are to be interpreted as encompassing the entire spirit and scope of the invention.